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Sliding-mode Control of Four Wheel Steering Systems

Lei Tan, Shuyou Yu
State Key Laboratory of Automotive Simulation and Control
Department of Control Science and Engineering
Jilin University
tanlei15@mails.jlu.edu.cn, shuyou@jlu.edu.cn

Yang Guo, Hong Chen
State Key Laboratory of Automotive Simulation and Control
Department of Control Science and Engineering
Jilin University
guoyang15@mails.jlu.edu.cn, chenh@jlu.edu.cn

***Abstract* - In this paper, a sliding mode control of active four-wheel steering systems is proposed in order to improve vehicle handling stability. An integral time-variant sliding surface is adopted to eliminate steady state errors, and a smooth function is used to alleviate the chattering effect. Dynamic responses of the front-wheel steering vehicles, the active four-wheel steering vehicles with LQR control and the active four-wheel steering vehicles with sliding mode control were compared. Simulation results show that the sliding mode control can track the ideal reference model and resist external disturbances.**

Index-Terms - Four-wheel steering; Handing stability; Sliding mode control; Chattering .

I. INTRODUCTION

The vehicle electrification and intelligence are new directions of modern automobiles, in which the steering system is developed to determine the active safety. Four-wheel steering (4WS) system is a key technology of vehicle chassis system [1].

The development of 4WS is roughly divided into two stages: active rear wheel steering and active four-wheel steering. The active rear wheel steering only controls the rear slip angles, and improves maneuver stability to some extent. Thus, it has to cooperate with other control strategies for the purpose of getting better performance. In order to obtain the ideal steering performance, active rear wheel steering systems based on μ-synthesis robust control [2], H_∞ control [3], sliding mode control [4], and model predictive control [5] are developed, respectively. Meanwhile, both yaw stability control and electronic stability control systems have the properties such as low consumptions, increase understeer or decrease oversteer. Thus, it caused serious threaten to the development of 4WS systems.

The 4WS systems is wildly discussed while steer-by-wire (SBW) technology is developed, which can effectively solve the problem of the active rear steering, that is, to reduce the changes of the steady-state gain of the yaw rate and lateral acceleration gain of vehicles caused by the change of vehicle speed and front wheel angle of the active rear steering. Vehicle status in running is measured by automotive body sensors. With the measurement, yaw rate and sideslip angle are calculated by the electric control unit (ECU). Compared them with an ideal reference model, the front and rear angles are given by controller so as to keep driving experience, improve cornering sensitivity, and enhance driving stability [6]. Thus, the development of steer-by-wire technology prompts the conduct of more research on the active 4WS systems. H_∞ controllers are designed in [7-8] which improve steering performance of vehicles, reduce the influence of parameter perturbations and external disturbances. Active 4WS systems with fuzzy control are proposed in [9-10], which are compared with front wheel steering (FWS) and proportional four-wheel steering systems. Optimal control scheme is adopted to solve the active 4WS control problems so as to achieve better robustness [11-12].

Sliding mode control is applied to the active 4WS systems [13-14] which can track the ideal model and enhance driving stability. Furthermore, 4WS system of vehicle is an uncertain, highly nonlinear, and complex system. Global sliding mode control in [15-16] is adopted to reduce the effect of uncertainties, and to eliminate the approaching mode.

In this paper, a global sliding-mode control is designed for 4WS systems. The chattering effect can be alleviated by replacing the sign function with a smooth function. In general, the boundary layer approach leads to steady state errors in the present of model uncertainties and external disturbances. An integral sliding surface is adopted in order to eliminate steady state errors. In the end, the active 4WS vehicles with the sliding mode control are compared with both the active 4WS vehicles with linear quadratic regulator control and the FWS vehicles.

II. PROBLEM SETUP

A. Model of active four wheel steering system

For steering system, the lateral acceleration and yaw rate of the vehicle include parameters such as the mass, yaw moment of inertia, tire cornering stiffness etc. As shown in Fig.1, the vehicle model is simplified to lateral and sideways movements which proved two degree of freedom active 4WS model. The model ensures accurately the basic characteristic during steering, and reduces the complexity of the control algorithm [1].

Assumptions are made in order to obtain the model of two degree of freedom active 4WS system:

- The influence of suspension and steering mechanism is ignored, that is, the vehicle just do longitudinal motion at a constant speed on the smooth ground;
- The steering angle of the front and rear wheels are treated as input;
- The tire side slip characteristics are in its linear range;
- The effect of the aligning torque is ignored;

- The left and right wheels have the same feature, and the front and rear wheels are replaced by the wheels with a double-cornering stiffness;
- Lateral acceleration of the vehicle is less than 0.4 g, and the wheel steering angle is less than 4°;
- Air resistance has no effect.

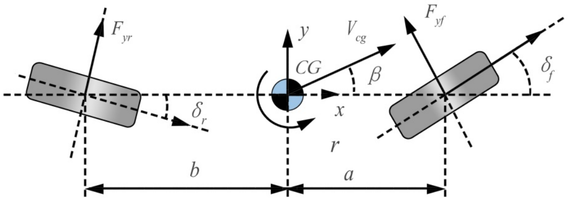


Fig. 1. Two degree of freedom active four-wheel steering model.

Dynamics of two degree of freedom active 4WS system are

$$mv(\dot{\beta}+\gamma)=-(k_f+k_r)\beta-\frac{\gamma}{v}\left(ak_f-bk_r\right)+k_f\delta_f+k_r\delta_r+d$$
$$I_z\dot{\gamma}=-(ak_f-bk_r)\beta-\frac{\gamma}{v}\left(a^2k_f-b^2k_r\right)+ak_f\delta_f-bk_r\delta_r+l_wd \tag{1}$$

where a and b are the distances from centroid to the front and rear axles, k_f and k_r are the front and rear wheels' cornering stiffness, β is sideslip angle, γ is yaw rate in vehicle body, v is centroid longitudinal velocity, δ_f and δ_r are the steering angle of the front and rear wheels, I_z is yaw moment of inertia, l_w is the distance that the disturbances acted on the vehicle to the centroid of the vehicle, m is the mass of the full vehicle, and d is the exogenous disturbance.

Define $x=[\beta \quad \gamma]^T$ as the state variable, $u=\left[\delta_f \quad \delta_r\right]^T$ is the control input, and assume that both β and γ can be measured. The state space equation of the systems is

$$\dot{x}=Ax+Bu+Ed \tag{2}$$

where

$$A=\begin{bmatrix}-\dfrac{k_f+k_r}{mv} & \dfrac{bk_r-ak_f}{mv^2}-1\\ \dfrac{bk_r-ak_f}{I_z} & -\dfrac{a^2k_f+b^2k_r}{I_zv}\end{bmatrix},$$
$$B=\begin{bmatrix}\dfrac{k_f}{mv} & \dfrac{k_r}{mv}\\ \dfrac{ak_f}{I_z} & -\dfrac{bk_r}{I_z}\end{bmatrix},\ E=\begin{bmatrix}\dfrac{1}{mv}\\ \dfrac{l_w}{I_z}\end{bmatrix}.$$

B. Reference model

The ideal steering behaviour of active 4WS system is to ensure zero sideslip and to keep the feeling of driving same as the traditional FWS vehicles. In other words, the ideal sideslip angle is zero, the ideal yaw rate is the same as the traditional FWS vehicles, and the steady-state gain is achieved by a first order inertia link. Define $x_d=\left[\beta^* \quad \gamma^*\right]^T$ as the state variable and $u_d=\delta_f^*$ is the input of the reference model, respectively, where β^* is the ideal side slip angle with the steady-state gain k_β, γ^* the ideal yaw rate with the steady-state gain k_h. Then, the reference model of vehicles with a 4WS systems is [17]

$$\dot{x}_d=A_dx_d+B_du_d \tag{3}$$

where $A_d=\begin{bmatrix}-\dfrac{1}{\tau_\beta} & 0\\ 0 & -\dfrac{1}{\tau_\beta}\end{bmatrix}$, $\quad B_d=\begin{bmatrix}\dfrac{k_\beta}{\tau_\beta}\\ \dfrac{k_h}{\tau_\gamma}\end{bmatrix}$, $\quad k_h=\dfrac{1}{1+kv^2}\cdot\dfrac{v}{L}$

and $k_\beta=0$. The time constants of the first-order inertia link are $\tau_\beta=\tau_\gamma$ =0.1 s in this paper.

C. Problem formulation

Define the state of the error systems

$$e=x_d-x=[\beta^*-\beta \quad \gamma^*-\gamma]^T, \tag{4}$$

Combing Eq.(2) with Eq.(3), the dynamics of the error systems can be written as

$$\begin{aligned}\dot{e}&=A_dx_d+B_du_d-(Ax+Bu+Ed)\\&=A_de+\left(A_d-A\right)x+B_du_d-Bu-Ed\end{aligned} \tag{5}$$

The following problem is considered in this paper:

Design a sliding mode control to regulate the steering angle of the front and rear wheels so as to:

(a) make sure that the dynamics of the active 4WS system (2) to follow the dynamics of the reference model (3),

(b) reduce the effect of disturbances or perturbations.

In order to design the sliding mode controller, some technical assumptions are made:

Assumption 1: The state of system (5) can be measured.

Assumption 2: The matrix B is invertible.

Assumption 3: $\text{Rank}(B)=\text{Rank}(B,E)$.

Assumption 3 which is named as the matching condition [18], means that the external disturbances share the same channel with the control input.

III. Sliding Mode Control

Define the sliding surface as [16], [19]:

$$S=e+\Psi\int_0^t e(\tau)d\tau+\theta(t). \tag{6}$$

where $\theta(t)=me^{-nt}$ with $0<n\in R^1$ and $m=-e(0)\in R^2$ is a time-varying item. Note that $m=-e(0)$ guarantees that $S(0)=0$, i.e., the system state is on the sliding surface at the initial time instant.

Suppose that the initial state of the system under control is $x(0)=0$, then $m=-e(0)=-x_d(0)$.

A. The equivalent control law

The derivative of S is

$$
\begin{aligned}
\dot{S} &= \dot{e} + \Psi e + \dot{\theta} \\
&= A_d e + \left(A_d - A\right)x + B_d u_d - Bu - Ed + \Psi e - mne^{-nt} \\
&= (A_d + \Psi)S - (A_d + \Psi)\Psi \int_0^t e(\tau)d\tau - (A_d + \Psi)me^{-nt} \\
&\quad + \left(A_d - A\right)x + B_d u_d - Bu - Ed - mne^{-nt}.
\end{aligned} \tag{7}
$$

The equivalent control law can be deduced by letting $d = 0$ and $\dot{S} = S = 0$,

$$
\begin{aligned}
u_{eq} = B^{-1}[&\left(A_d - A\right)x + B_d u_d - \left(A_d + \Psi\right)\Psi \int_0^t e(\tau)d\tau \\
&- (A_d + \Psi)me^{-nt} - mne^{-nt}].
\end{aligned} \tag{8}
$$

Ignoring the impact of the uncertainty, i.e., $d = 0$, the dynamics of the error systems on the sliding surface is

$$
\dot{S} = (A_d + \Psi)S. \tag{9}
$$

Choosing $\Psi = -A_d$ and $m = -e(0)$, the state of the error systems will stay on the sliding surface. Then the equivalent control law is simplified as

$$
u_{eq} = B^{-1}[\left(A_d - A\right)x + B_d u_d - mne^{-nt}]. \tag{10}
$$

B. *The robust control law*

An exponential reaching rate with time-varying switching gain [20-21] is adopted in this paper, which has the advantages of both the variable speed reaching law and classical exponential reaching law. This new type of reaching law guarantees that the sliding mode motion converges to the sliding surface, and the phenomenon of chattering is weakened effectively.

The new reaching law can be chosen as:

$$
\dot{s}_i = -\varepsilon_i \Gamma(s_i)\operatorname{sgn}(s_i) - \eta_i s_i, \quad i = 1, 2, \tag{11}
$$

where $\varepsilon_i > 0, \eta_i > 0,$ $\Gamma(\rho) = \dfrac{|\rho|}{|\rho| + \mu}$ with $\rho \in R^1$ is a time-varying item, μ is a smaller positive constant. Furthermore, $\Gamma(s_i) \approx 1$ while $|s_i| >> \mu$; $\Gamma(e_i)$ approaches to 0 while $|s_i| \to 0,$ c.f., Fig.2

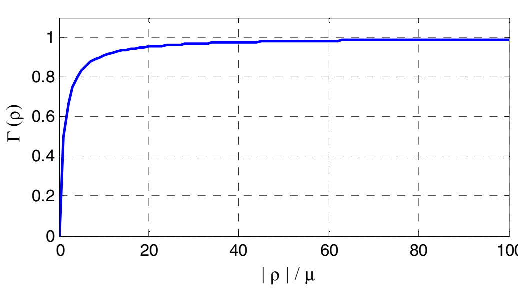


Fig.2 The function of $\Gamma\left(\rho\right)$

By abuse of notations, define

$$
\operatorname{sgn}(S) = \begin{bmatrix} \operatorname{sgn}(s_1) & 0 \\ 0 & \operatorname{sgn}(s_2) \end{bmatrix}, \quad \Gamma(s) = \begin{bmatrix} \Gamma(s_1) & 0 \\ 0 & \Gamma(s_2) \end{bmatrix}.
$$

Accordingly, choose the robust control law as

$$
u_{rob} = B^{-1}\left[M_1 S + M_2 \Gamma(s)\operatorname{sgn}(S)\right] \tag{12}
$$

where $M_1 = \begin{bmatrix} -\eta_1 & 0 \\ 0 & -\eta_2 \end{bmatrix}$, $M_2 = \begin{bmatrix} -\varepsilon_1 & 0 \\ 0 & -\varepsilon_2 \end{bmatrix}$, $\eta_i > 0$ and $\varepsilon_1 > 0$ for $i = 1, 2.$

Therefore, sliding mode control of active 4WS systems with respect to disturbance can be expressed as follows

$$
\begin{aligned}
u &= u_{eq} + u_{rob} \\
&= B^{-1}[\left(A_d - A\right)x + B_d u_d - mne^{-nt}] + B^{-1}[M_1 S + M_2 \Gamma(s)\operatorname{sgn}(S)].
\end{aligned} \tag{13}
$$

The according sliding motion with $u = u_{eq} + u_{rob}$ is

$$
\begin{aligned}
\dot{S} &= \dot{e} + \Psi e + \dot{\theta} \\
&= (A_d - A)x + B_d u_d - B\left(u_{eq} + u_{rob}\right) - Ed - mne^{-nt} \\
&= \begin{bmatrix} -\eta_1 & 0 \\ 0 & -\eta_2 \end{bmatrix} S + \begin{bmatrix} -\varepsilon_1 \Gamma(s_1)\operatorname{sgn}(s_1) \\ -\varepsilon_2 \Gamma(s_2)\operatorname{sgn}(s_2) \end{bmatrix} - Ed
\end{aligned}
$$

C. *Analysis of robustness*

For convenience sake, denote $Ed = \left[d_1 \quad d_2\right]^T$. Define the Lyapunov function

$$
V = \frac{1}{2} S^T S. \tag{14}
$$

Then,

$$
\begin{aligned}
\dot{V} &= S^T S \\
&= [s_1 \quad s_2]\left(\begin{bmatrix} -\eta_1 s_1 - \varepsilon_1 \Gamma(s_1)\operatorname{sgn}(s_1) \\ -\eta_2 s_2 - \varepsilon_2 \Gamma(s_2)\operatorname{sgn}(s_2) \end{bmatrix} - Ed\right) \\
&= -\varepsilon_1 \Gamma(s_1)s_1 \operatorname{sgn}(s_1) - \varepsilon_2 s_2 \Gamma(s_2)\operatorname{sgn}(s_2) \\
&\quad - \eta_1 s_1^2 - \eta_2 s_2^2 - \left([s_1 \quad s_2]Ed\right) \\
&\le -\eta_1 s_1^2 - \eta_2 s_2^2 - |s_1|\left(\varepsilon_1 \Gamma(s_1) - |d_1|\right) - |s_2|\left(\varepsilon_2 \Gamma(s_2) - |d_2|\right).
\end{aligned} \tag{15}
$$

For all $|s_i| > 10\mu,$ $\Gamma(s_i) > 0.9,$ $i = 1, 2,$ and

$$
\dot{V} = -\eta_1 s_1^2 - \eta_2 s_2^2 - |s_1|\left(0.9\varepsilon_1 - |d_1|\right) - |s_2|\left(0.9\varepsilon_2 - |d_2|\right) \tag{16}
$$

Choose ε_i such that $\varepsilon_i > |d_i| / 0.9,$ $i = 1, 2,$ then $\dot{V} < 0,$ i.e., the system under control with respect to disturbances will approach asymptotically the zone, $|s_i| < 10\mu,$ $i = 1, 2,$ which is close to the sliding surface.

D. *Chattering suppression*

The implementation of sliding mode control is often irritated by high frequency oscillations known as “chattering” in system outputs issued by dynamics from actuators and sensors ignored in system modelling. It is important to reduce chattering for the purpose of reducing energy consumption, resisting external disturbances and un-modelled high-frequency dynamics. Thus, in this paper the sign function $\operatorname{sgn}(s_i)$ in [11] is replaced by the smooth function [22] $con(s_i)$ shown in Fig. 3

$$
con(s_i) = \frac{s_i}{|s_i| + \varsigma} \tag{17}
$$

where the smoothing factor $0 < \varsigma \in R^1$ is a constant.

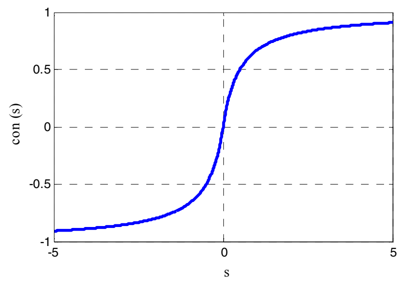


Fig. 3 The smooth function of $con(s)$

IV. SIMULATION

A nonlinear 8 DOF model is chosen as the simulation model [23] in which the Dugoff tire model is adopted [24]. The 8 DOF model can accurately reflect the dynamic of vehicle. The effectiveness of the sliding mode control is compared with the effectiveness of the traditional front-wheel steering vehicle and the active 4WS vehicle under LQR control based on steer-by-wire technology. Parameters of the vehicle are shown in Table I, and parameters of sliding mode controller (13) are set to $\eta_1 = 100$, $\eta_2 = 150$, $\varepsilon_1 = 100$, $\varepsilon_2 = 10$.

TABLE I
Vehicle parameters

Symbol	Parameter	Value
m	mass	1704.7 kg
a	centroid distance to the front axle	1.035 m
b	centroid distance to the rear axle	1.665 m
I_z	yaw rotational inertia	3048.1 $kg \cdot m^2$
k_f	the front wheel cornering stiffness	39515.0 N/rad
k_r	the rear wheel cornering stiffness	39515.0 N/rad

A. Straight line condition

The wind disturbances, which frequently occur to the vehicle during high speed travelling, can seriously affect handling and stability of vehicles. Bi-directional crosswind experiment of vehicles is carried out while vehicles are travelling at high speed on a straight line. In the simulation, the effect of aerodynamic drag, lift and pitching moment are neglected. Instead, only the aerodynamic side force, yaw and roll moments are considered. The vehicle speed is set to 30 m/s. The crosswind is acted at the rear of the centroid of the vehicle body, the crosswind-induced arm of a force is $l_w = -0.1$, the front-wheel angle is set to zero. The speed of the crosswind is altered from 15 m/s to −15 m/s exactly at the instant t = 1.5 s as shown in Fig. 4.

From Fig.5 (a), (b), and (c), the side slip angle, the yaw rate and the lateral displacement of the FWS vehicle are fluctuated considerably. Thus, the bi-directional crosswind has severe impact on the handling and stability of the vehicle. In contrast, the side slip angle, the yaw rate and the lateral displacement of the active 4WS vehicle with the sliding mode control remain small. There is no overshoot even the direction of the crosswind is suddenly changed. It can track the ideal model and hold a good body position. Compared with sliding mode control, LQR control only offers a little inherent robustness to some extent.

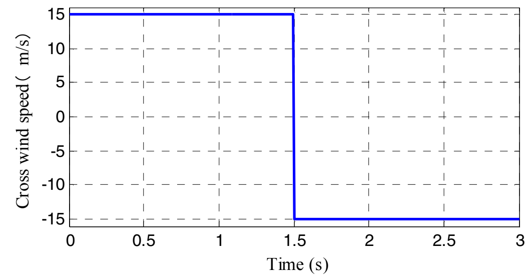
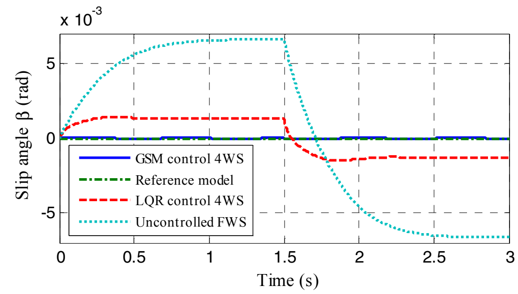
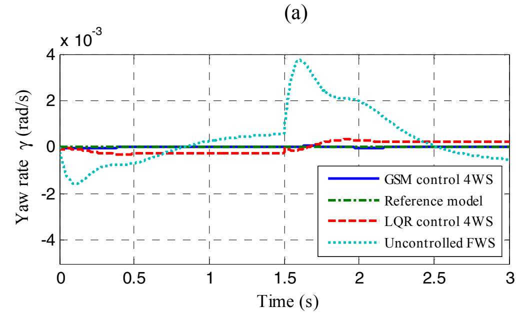


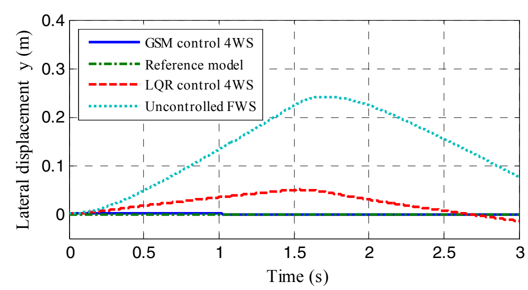
Fig. 4. Disturbance input of Two-way cross wind.



(a)



(b)



(c)

Fig. 5. Response curve of two-way crosswind.

B. The single lane change condition

Changing lanes is a movement from one lane to another on roads with two or more lanes in the same direction. In the process of lane change, dynamic characteristics as well as safety of vehicles are seriously affected by external interference. In this section, a single lane change manoeuvres with respect to a crosswind is carried out for vehicles travelling at high speeds. In simulation, the six components of aerodynamic force are chosen the same as the straight line condition. The vehicle speed is 30 m/s, the steering angle of the front wheel is sinusoidal with the angular frequency of 2.512 rad/s and the amplitude of 0.035, which is shown in

Fig.6. The crosswind with the speed of 15 m/s during the interval 2.5-5 s is acted at the rear of the centroid of the vehicle body, the crosswind-induced arm of a force is $l_w = -0.1$.

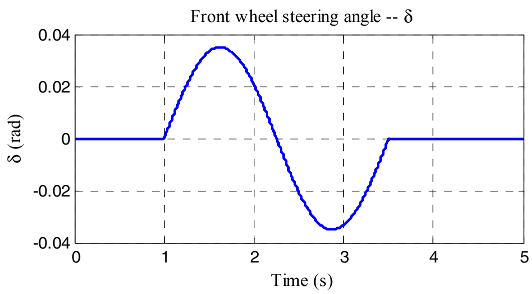


Fig. 6. Input of the front wheel rotation in single lane change condition.

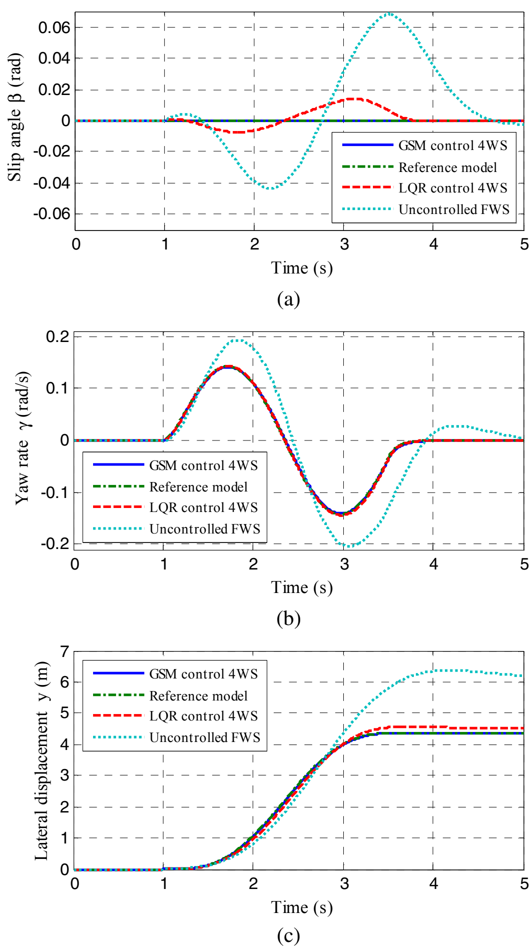


Fig. 7. Response curve of two-way crosswind.

As shown in Fig.7 (a), the amplitude of the side slip angle of either the FWS vehicle or the active 4WS vehicle with LQR control has a significant change. However, the side slip angle of the active 4WS vehicle with sliding mode control is kept constantly close to zero in the process of the single lane change, i.e., the attitude of the vehicle is well maintained. From Fig.7 (b), it can be seen that, considering crosswind, both the active 4WS vehicle with sliding mode control and the active 4WS vehicle with LQR control can track the yaw rate of the ideal model with higher precision compared to the FWS vehicle, and guarantee stability of vehicle. The active 4WS vehicle with sliding mode control has better performance than the active 4WS vehicle with LQR control. Fig.7 (c) shows that there exists a certain deviation between the trajectory of the FWS vehicle and the ideal one. The accuracy of tracking of the active 4WS vehicle with sliding mode control is higher than the active 4WS vehicle with LQR control. In a crosswind environment, only the active 4WS vehicle with sliding mode control can perfectly accomplish the task of the single lane change.

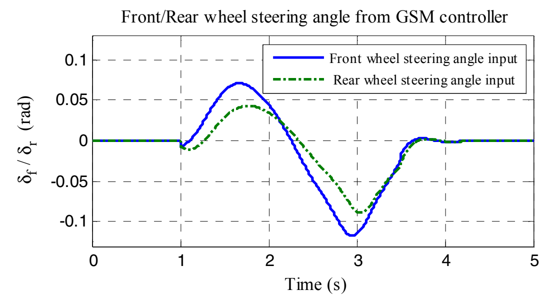


Fig. 8. The output of the global sliding model.

As shown in Fig.8, the front and rear angles of the wheel of the active 4WS vehicle with sliding model control lie in a reasonable range.

V. Conclusions

Based on steer-by-wire technology, this paper designed a sliding-mode controller for the active four-wheel steering system, which both side slip angle and yaw rate can trace the given reference trajectories. Simulation results show that, compared with the front wheel steering and active four-wheel steering vehicle based on linear quadratic control law, the proposed sliding-mode control strategy can effectively deal with disturbances, and improve handling stability of the vehicle.

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